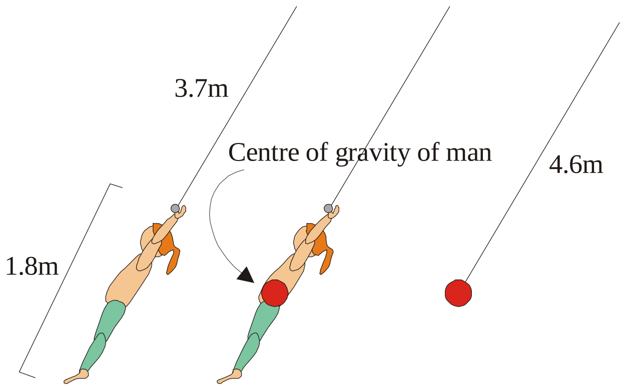
Investigation 1: Energy changes in the swing

Assumptions

For this investigation, we will model the trapeze and performer as a simple pendulum (i.e. a mass on the end of a light inextensible string). We will assume that the flyer does not try to move his body, and hangs in line with the cable (obviously not the case, but we can approximate to this, in such a simple investigation).



As the mass of the man is spread out along his length, we will have to approximate, and model the man as a particle, positioned at the man's centre of mass, which we will assume to be halfway down his body. The man and trapeze is therefore modelled as a mass of 74kg (70kg for man, and 4kg for bar), at a distance 4.6m from top of cable (3.7m for cable, and extra 0.9m for half man's length).

We will also ignore air resistance.

Energy

There are three types of energy involved in a trapeze swing: kinetic, potential and chemical (in the man). As in this investigation we are not allowing the man to move himself, we can ignore the third.

When the man starts from the board, he is initially not moving, and so he has his maximum potential energy (PE), given by:

$$PE = m \times g \times h$$

(m = mass, g = gravitational constant, h = height)

Once he has swung to his lowest point, gravity has accelerated him to his maximum speed and therefore his maximum kinetic energy (KE) which is given by:

$$KE = 1/2 \times m \times v^2$$

(m = mass, v = velocity (in this instance this is equal to speed))

All the kinetic energy that he now has has been converted from his potential energy as he swung down to the bottom. At the bottom, he therefore has his lowest amount of potential energy.

We will assume the bottom of his swing to be the point of reference for all the energy calculations (i.e. O), as we are only interested in ‘change in height’. Therefore at the bottom he has ZERO potential energy and ALL the PE he had in the first place is now KE.

We can therefore say that:

$$PE = KE$$

We can then substitute the formulae for PE and KE:

$$mgh = {}^{1}/_{2}mv^2$$

Cancelling the m and rearranging gives:

$$v^2 = 2gh$$

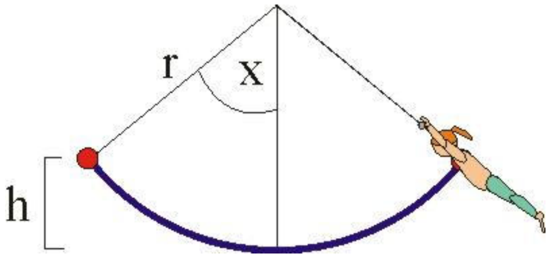
From this expression, we can find his maximum speed, or indeed his speed at any point on his swing, just by knowing h, the distance he has moved VERTICALLY from the board.

By symmetry, on the other side of his swing, his KE will turn back to PE, and if no energy is lost from the system, he should end up on the other side at the same height he started at.

In practice, he loses energy to heat (the air around him heats up slightly due to friction with his body). There is therefore less energy available to swing him back up to the same height, and, if he does not use some of his own chemical energy in his body to 'top up' the energy of his swing (using a more advanced swing technique), he will swing slower and slower and eventually stop.

Calculating Maximum Speed:

Let us now find the maximum speed of a man on a trapeze, modelled as above:



$$v^2 = 2gh$$

$g = 9.8$ (a constant)
$h = 1.5$m on a standard rig

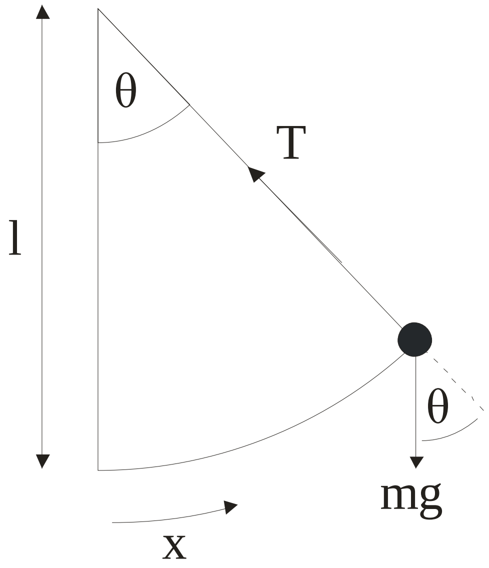
$$v^2 = 2 \times 9.8 \times 1.5$$

$$v^2 = 5.4ms^{-1}$$

Using Simple Harmonic Motion, to find time period of swing:

We must first prove than SHM is a valid way to model this problem:

F = ma along the path of pendulum (positive for displacements to right)



$$- mg\sin\theta = m\ddot{x}$$

$$\text{But } x = l\theta \,;\ \ddot{x} = l\ddot{\theta}$$

$$-mg\sin\theta = m\ddot{\theta}$$

$$\ddot{\theta} = \frac{-g}{l}\sin\theta$$

$$\text{For small } \theta\text{: } \theta \approx \sin\theta$$

$$\frac{\sin\theta}{\theta} \xrightarrow{\theta\to 0} 1$$

$$\therefore \ddot{\theta} \approx \frac{-g}{l}\theta$$

This is in the form $\ddot{x} = \alpha^2 x$ (where in this case $\omega^2 = \sqrt{\frac{g}{l}}$) which is the condition for SHM.

Therefore we must approximate $\sin\theta = \theta$ in order to obtain expression $\theta = -\omega^2\theta$

In our case θ is approx. 0.83 radians, so $\sin\theta = 0.73$, which is (fairly) close to θ, so we are justified in making this approximation.

Therefore time period (time from one point in swing, to same point in next swing) is given by:

$$T = \frac{2\pi}{\alpha} \text{ where } \omega = \frac{g}{l}$$

Therefore in this case:

$$T = 2\pi\frac{l}{g}$$

(l = 4.6m, g = 9.8)

$$T = 2 \times \pi \times 4.6/9.8$$

$$T = 4.3\text{s}$$